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MATHEMATICAL MODEL EFFICIENT POSITION GENERATOR FOR LINEAR WIRELESS SENSOR NETWORKS

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ABSTRACT
The mathematical sciences have always played a preponderant role in the reflection, development, and implementation of technological models and mechanisms. A Linear Wireless Sensor Network (LWSN) is a collection of sensors arranged in linear geometry. We can therefore represent a linear WSN as being a set of algebraic objects, arranged according to a very precise topology, with characteristics such as the position (coordinates in space) the transmission range, the cardinality of the network (number of nodes that compose the network) etc...
Several types of topology have been proposed for linear RCSFs in order to provide them with better organization. One of them assumes a strictly linear topology, in which we note the existence of a single linear line extending from the sink.
In this article we propose a new kind of topology called k-variant redundant where the redundancy factor k is a variable. This topology is based on a model generating the Euclidean positions of the nodes in increasing ways, which will make it possible to vary the redundancy factor k. The proposed topology model increases network availability especially in areas close to the sink, reducing the transmission effort (energy) at the nodes as one approaches the sink.
The proposed model was compared with the existing topologies, namely the k-redundant topologies and the random placement model. The results obtained showed a longer lifespan for the proposed model, which makes it more efficient in terms of energy consumption.

KEYWORDS: applied-linear-algebra, linear-wireless-sensor-network, topology-generator.

1. INTRODUCTION
Linear algebra is a universal language that is used to describe many phenomena in mechanics, electronics, economics, computer science, etc. The interest of such a field in the real world is to study the properties and movements of objects (physical or logical) in a given geometric space. It thus makes it possible to model a physical environment in linear form, as a consequence of which it can solve a set of problems existing in such structures.
Linear WSNs are a collection of sensors arranged in a linear geometry [1] [2] [3]. We can represent a linear WSN as a set of algebraic objects with characteristics like position (coordinates in space), arranged in a specific topology. Several types of topology have been proposed for linear WSNs in order to provide them with better organization. The topology of a linear WSN depends on the environment in which it is deployed and its application. LWSNs are essentially divided into two categories: First, we have networks with strictly linear topologies. In this type of network, we note the existence of a single linear line starting from the sink. We also note a total absence of junction zones. Secondly, we have linear topology networks with junction areas. In these types of networks, we note the existence of several linear lines, some of which form crossing points called junctions [2].
An LWSN can then be defined as a set of physical objects exhibiting algebraic properties and characteristics. As a result, algebraic structures and properties such as groups, rings, families, vectors, mappings etc. become applicable to a linear wireless sensor network.
In this article, we propose a new kind of topology called redundant k-variant in which the redundancy factor k is a variable. The proposed model makes it possible to generate the Euclidean positions of the nodes in an increasing way which will make it possible to vary the redundancy factor k. Such a topology increases network availability

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especially in areas close to the sink by reducing the transmission effort (energy) at the nodes as one approaches the sink.
The article will be organized as follows: In the first part, a state of the art on topology models in LWSN will be done. Part 2 will be devoted to the presentation of our model. In the third part, we will present the results of the simulations carried out. Finally, the last part will concern the conclusion and some perspectives.

2. RELATED WORK

The placement of the nodes [7] is one of the factors that can lead to non-uniform energy consumption and therefore shorten the lifetime of the network. Therefore, the nodes must be placed strategically to extend the lifetime of the network. Many existing node placement schemes offer random, uniform, and non-uniform placement techniques. Unlike most WSN applications, random node distribution may not be feasible for large scale applications.
In [5], the authors first study the problem of linear placement of sensors in order to maximize their lifetime. Regarding the simple equal-distance placement scheme, the authors show that the result based on the ideal power model is not efficient compared to that of a realistic power model derived from Tmote Sky sensors [6]. The authors also study equal power placement schemes and formulate the problem as a MILP (Linear Mixed Integer Programming) problem. They thus propose two efficient heuristic systems of placement, with which the lifespan of the sensors is considerably improved.
In [7], the authors conduct a detailed study on the impact of node placement in WSNs. However, it becomes necessary to do a more in-depth study when it comes to networks with linear topologies because the geometric specifications are quite different. Several techniques for placing nodes for WSNs exist in the literature. One of these is to randomly place nodes in the deployment region. On the other hand, such a deployment does not meet design considerations like coverage, connectivity, fault tolerance, etc. This non-compliance becomes even more important when it comes to a linear network.
In [4], the authors propose an incremental random node placement algorithm (IRNP) based on the work carried out in [8]. To ensure coverage, they assume linear sequential placement. The first node is positioned at the edge of the segment, and the horizontal location of the next node is randomly determined. To ensure network connection, the next node is placed in the new location if it is within communication range of the previously deployed node. The procedure is repeated recursively until all nodes are placed.
One of the consequences of random placement is the non-standardization of the network, which can lead to overconsumption of energy. Faced with this work has been proposed going in the direction of proposing deployment techniques with uniform placement of nodes [7]. Uniform placement deployment aims to distribute the nodes evenly at equal distances (i.e., di = di + 1). Such schemes are viable for pre-planned and controlled deployment. For example, with a segment length (L) and a given number of nodes (N), the distance (d) between nodes in linear sequential uniform deployment will be: d = L/N. This kind of topology also exists under the name of k-redundant topology.
Uniform placement also suffers from overconsumption of energy. This is because nodes close to the sink must transmit data from distant sensors in addition to their own data. One solution is to position the sink in the middle this will be able to balance the energy consumption by reducing the range of the multi-hop communication. However, the nodes closest to the sink will still deplete their energy quickly as they have to transmit additional data from distant sensors. Therefore, a non-uniform node pattern remains the only solution that can balance the load and therefore extend the life of the linear network.

Placement of nodes with linearly decreasing distance in LWSNs
We have already seen the inability of random and uniform patterns to balance grid energy consumption. In [4], the authors present (LDD), a non-uniform node placement scheme in which the distance between nodes is not uniform. To balance power consumption, the linear decreasing scheme uses a decreasing distance function that reduces the distance between nodes to the sink (Figure 1). The goal of (LDD) is to distribute the long-range transmission load on remote sensors and the multi-hop data transfer overhead on neighboring nodes. (LDD) find the distance between nodes based on equation (1):

$$d_i = \frac{L}{i \times \sum_{i=0}^{N} \frac{1}{i}} \quad (1)$$

where d_i < Transmission range.

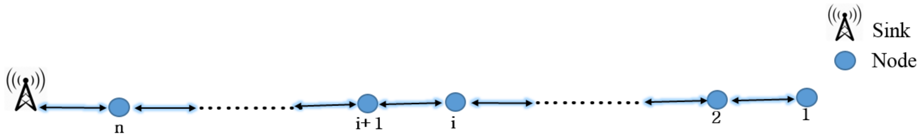

Figure 1: Sequential placement with sink on board

In sequential placement with sink on board, it is necessary to have a sufficient number of nodes (N) for a linear segment L, in other words the network must be convergent over the entire linear segment in order to be able to extend the lifetime of the network. If this is not the case there is a risk of having two nearby nodes separated by a distance which is greater than their transmission range.
With (LDD), the authors propose a second method consisting to place the sink in the middle (figure 2). This solution helps to regulate the constraint related to the range, and also increases the life of the network by balancing the consumption of energy. Equation (1) becomes equation (2):

$$d_i = \frac{\frac{L}{2}}{i \times \sum_{i=0}^{N} \frac{1}{i}} \quad (2)$$

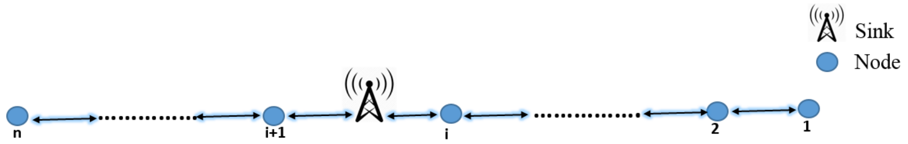

Figure 2: Sequential placement with sink in the middle

K-redundant topology
In [9], the authors propose a topology based on a k-redundant architecture in which each node of the network has k neighbors in the direction of the sink and in the opposite direction, if it is a strictly linear network each node a, at most, 2 × k neighbors and at least k neighbors (Figure 3). In this type of topology, the availability of the WSN strongly depends on the value of k, the greater this value, the better the network is in terms of availability.

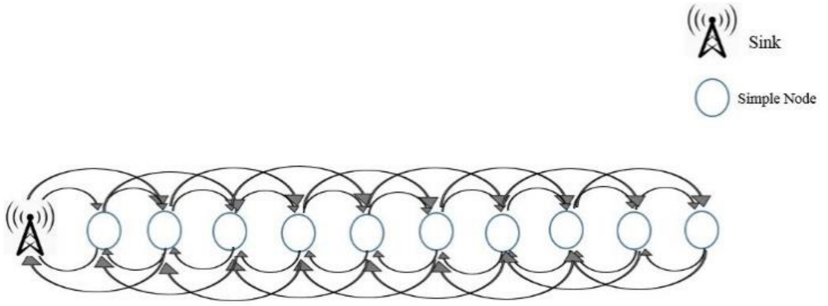

Figure 3: WSN: 2-redundant

Reminder of some mathematical concepts
In this part we recall the basic elementary algebraic notions commonly used in mathematics and which will be used to understand the following.

- **Set**: A set is a collection of objects, called elements. We write a set by curly braces {0; 1; 2; 3; …….} with each item separated by a semicolon. (The order in which the items appear is not important). In the following, we will denote by N the set of nodes belonging to the linear WSN. The clusters will be considered as subsets of the set N.
- **Set mappings**: A mappings f: $A \rightarrow B$ is a process which associates to any element a of A with a unique element ***f(a)*** of B. The set A is called the source set, the set B is called the set goal.

Positional deterministic model

In [2], the authors propose a model making it possible to determine the Euclidean position of a node with respect to the sink. The authors define a scoring system given by Table 1 that perfectly reflects the correlation that exists between a network of linear wireless sensors and a finite set of elements.

For any node n belonging to a cluster $\boldsymbol{C_x}$ of the network, its position $\boldsymbol{P_{n \to S}}$ (distance) with respect to the sink is given by equation (iii).

$$\forall n \in C_x; \quad P_{n\to S} = P_n^x + 2 \times \left(CJ_{C_x}\right) + \sum_{CS_i \in S_{C_x}} (C_{CS_i}) \quad (3)$$

Table 1. Table of notations

Notation	Meaning
$N = \{............\}$	The set of nodes in the WSN
$C = \{............\}$	The set of clusters in the WSN
P_n^x	The position of node n in cluster x
$F_{C_x} = \{......\}$	the set of Clusters (son of C_x) belonging to the line leading to the SINK
$J_{C_x} = \{......\} \subset F_{C_x}$	The set of junction clusters belonging to F_{C_x}
$S_{C_x} = \{......\} \subset F_{C_x}$	The set of simple clusters belonging to F_{C_x}
$J_{C_i} \cup S_{C_i} = F_{C_i}$	
C_{C_i}	the cardinality of the cluster C_i
C_{CJ_i}	the cardinality of the junction cluster CJ_i
C_{CS_i}	the cardinality of the simple cluster CS_i
CJ_{c_x}	the cardinality of the set J_{C_x}

3. MATHEMATICAL MODEL EFFICIENT GENERATOR OF POSITIONS FOR A LINEAR WSN

In this part we present the mathematical model efficient generator of positions for a linear WSN proposed in this work.

Postulates and Notation:

- ✓ We consider a homogeneous network of strictly linear wireless sensors [3];
- ✓ We denote by $\boldsymbol{d}$ the distance from the linear axis of the network;
- ✓ We denote by $\boldsymbol{C_r}$ the cardinality of the network (the number of nodes that make up the network);
- ✓ We denote by $\boldsymbol{p}$ the maximum range of a node: this value is a constant, given the homogeneity of the network.

Convergence conditions

- The number of nodes with their transmission range must cover the entire linear distance of the network, equation (4):

$$p \times C_r \geq d \Rightarrow p \geq \frac{d}{C_r} \qquad (4)$$

- For any LWSN satisfying the convergence condition, there exists a non-zero real k_f such that the network is kf-redundant, equation (5).

$$k_f \geq \frac{p \times C_r}{d} \qquad (5)$$

Mathematical model

We define a map $\boldsymbol{f}$ given by equation (6):

$$If \;\; p = \frac{d}{C_r}: \qquad f(C_r, d, p) = \frac{p \times C_r}{d} = k = 1 \quad (6)$$

In this case the network is 1-redundant.

$$If\ p > \frac{d}{C_r}:\ f: N^* \times N^* \times N^* \rightarrow M_{C_r,2}(N^*): \forall (C_r, d, p) \in N^{*3} \text{ we associate } f(C_r, d, p)\ |$$

$$f(C_r, d, p) = \begin{cases} G_1\left(\frac{p}{k}\right) + G_2\left(\frac{p}{k-1}\right) = d \\ G_1 + G_2 = C_r \quad k \in \mathrm{N}^*\backslash\{1\} \end{cases} with\ (G_1,\ G_2) \in [2, C_r - 2]^2 \quad (7)$$

- ***k*** is the maximum network redundancy factor, ***G1*** and ***G2*** are two groups of nodes having respectively *k* and *k-1* as redundancy factor.
- **Theorem:** Let G1 and G2 be two groups of nodes of respective redundancy factor *k* and *k-1*. The nodes of G1 will be distanced by $\frac{p}{k}$ from each other, those of G2 will be distanced by $\frac{p}{k-1}$.
- We denote by $M_{C_r,2}(N^*)$ the position matrix containing the positions of all the nodes of the network.
- The generated matrix $M_{C_r,2}(N^*)$ is defined, on each row, by the pair (Id_{node}, D_{sup}) with Id_{node} is the identifier of the node and D_{sup} the distance which separates it from its neighbor greater than one jump (in the direction of the sink).

Applying the model

We consider the topology of figure 4 defined by: *Cr* = 50 nodes, *p* = 10 m, *d* = 200 m.

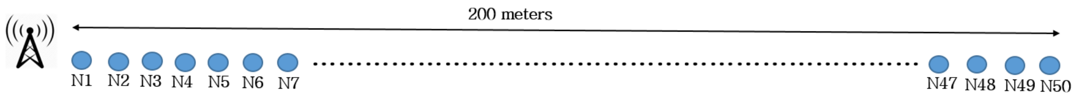

Figure 4: Topology

We first check the convergence condition:

We have $10 \geq \frac{200}{50}$ so $p \geq \frac{d}{C_r}$

$$f(50{,}200{,}10) = \begin{cases} G_1\left(\frac{10}{k}\right) + G_2\left(\frac{10}{k-1}\right) = 200 \\ G_1 + G_2 = 50 \quad k \in N^*\backslash\{1\} \end{cases} \text{with } (G_1, G_2) \in [2;\ 48]^2 \quad (8)$$

We then look for a k belonging to [2; *p*] for which the solutions *G1* and *G2* exist for *f*.

- For *k* = 2: the solutions G1 and G2 do not exist in [2; 48]
- For $k = 3$:

$$f(50{,}200{,}10) = \begin{cases} G_1\left(\frac{10}{3}\right) + G_2\left(\frac{10}{2}\right) = 200 \\ G_1 + G_2 = 50 \end{cases} \quad (9)$$

We find G_1=30 and G_2=20 as solutions for f.

We have a 3-redundant node group of 30 and a 2-redundant node group. The nodes of group G_1 are distanced by $\frac{p}{k} = \frac{10}{3}$ meters those of group G_2 are distanced by $\frac{p}{k-1} = \frac{10}{2} = 5$ meters (Figure 5).

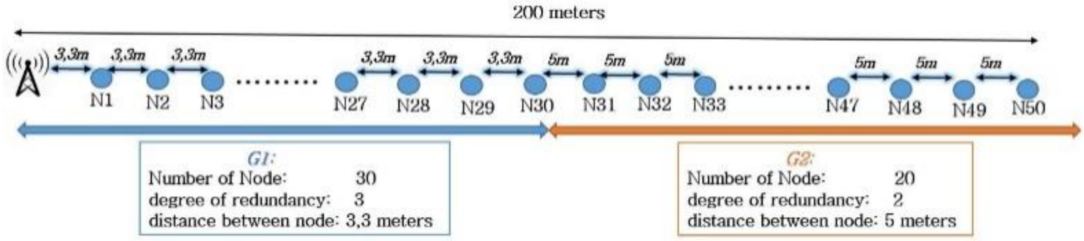

Figure 5: Topology obtained

The generated position matrix $\boldsymbol{M_{50,2}}(\boldsymbol{N^*})$ obtained is given below:

$$M_{50,2} = \begin{pmatrix} N1 & 3.3m \\ N2 & 3.3m \\ N3 & 3.3m \\ \vdots & \vdots \\ N29 & 3.3m \\ N30 & 3.3m \\ N31 & 5m \\ N32 & 5m \\ N33 & 5m \\ \vdots & \vdots \\ N48 & 5m \\ N49 & 5m \\ N50 & 5m \end{pmatrix}$$

4. SIMULATIONS AND RESULTS

We consider the network defined by the topology of figure 4. The proposed position model is compared to a fixed-redundant topology defined by the same characteristics (cardinality, range, linear distance) in figure 6 and with a random positioning topology with the same characteristics at the nodes. We define the lifetime of an LWSN as the time it takes until the last neighbor of the sink dies. This duration constitutes the time beyond which no data can be transmitted to the base station.

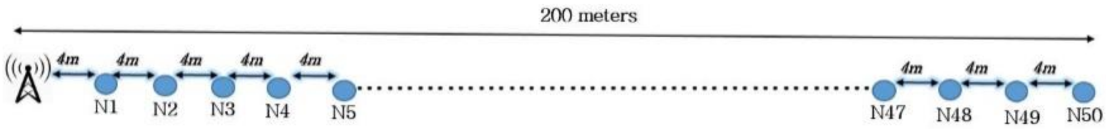

Figure 6: 2.5-redundant topology

The degree k_f of the k_{fixed}-redundant topology is given by equation (10):

$$k_f = \frac{10 \times 50}{200} = 2.5 \qquad (10)$$

We denote by D_n the distance between the nodes given by equation (11):

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$$Dn = \frac{p}{k_f} = \frac{10}{2.5} = 4\ m \qquad (11)$$

All nodes initially have an energy reserve equal to 18,720 joules (Castalia model): $Ei = 18720\ joules$.
We measured the time it took until the last neighbor of the sink extinguished. We carried out several series of transmissions on the three models of topologies taken. Figure 7 gives us the lifetime of the three topologies on each transmission round. The results obtained show a lifetime greater than the level of the proposed model.

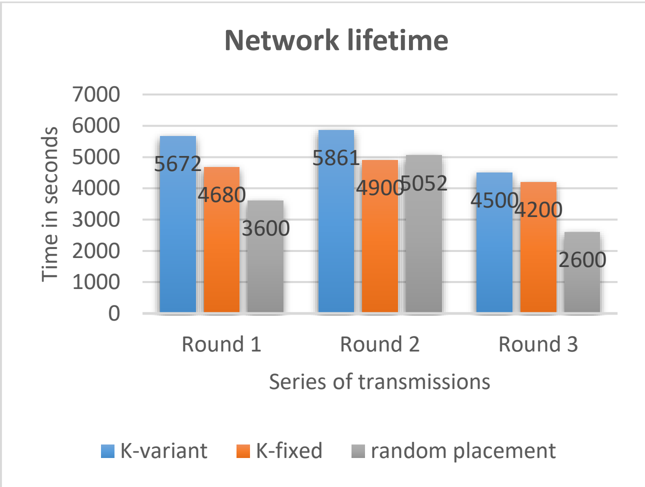

Figure 7: network lifetime per transmission series

5. CONCLUSION AND PERSPECTIVES

In this paper we have proposed a mathematical model based on linear algebra. The topological structure of a linear wireless sensor network can be assimilated to a set of physical objects (sensor nodes) with characteristics such as the Euclidean position of the nodes, the transmission range, the cardinality of the network, etc. thus becomes possible to establish a correlation between a linear WSN and a finite set of elements. As a result, certain algebraic properties become applicable on a linear topology WSN.
The model proposed in this work allowed us to set up a new kind of topology called redundant k-variant in which the redundancy factor k is an increasing variable. The proposed topological model was compared with the existing topologies, namely the k-redundant topologies and the random placement model. The results obtained showed a longer lifetime for the proposed model, which makes it more efficient in terms of energy consumption. However, future work can be considered with other comparison criteria such as network load, data loss rate, latency etc.

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